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LOAD-DEFORMATION CHARACTERISTICS OF CONCRETE PRISMS WITH RECTILINEAR TRANSVERSE REINFORCEMENT

by

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1. INTRODUCTION

1.1 Introductory Remarks

The beneficial effects of transverse reinforcement on the strength and deformation characteristics of concrete have been recognized since the early days of reinforced concrete construction. A patent was granted to Thaddeus Hyatt in 1874 for the use of helically-wound flat bars with encased longitudinal rods in reinforced concrete members (1,2)*. However, this type of member did not find wide application until after the experiments of Considere at the turn of the century.

Hyatt had not limited the application of his idea to members of circular cross section. In fact, he recommended it even for I-shaped sections with I-shaped enclosed cores. Considere also started out recommending transverse reinforcement in all types of members, especially in continuous beams (3). However, his early tests on columns showed that, for equal weights of reinforcement, circular hoops were more than twice as effective for strength as a system of rectilinear transverse reinforcement (4). Consequently, Considere confined his research to circular hoops and helices. For rectangular sections with large length to width ratios, he investigated the use of a series of intersecting hoops. For beams, he used small diameter coils of helicoidal reinforcement in the compression zone rather than trying to utilize the stirrups in this capacity. Thus, the further development of transverse reinforcement in reinforced concrete construction was limited to hoops or helices.

The only type of transverse reinforcement which was involved in the extensive ACI Column Investigation of the 1930's was the helix which was referred to as "spiral" reinforcement, the columns having this type of reinforcement

* Numbers in parentheses refer to entries in the list of references.

being called "spiral columns". In the course of the ACI investigation, tests were carried out on concrete cylinders confined laterally by hydraulic pressure (similar to Considere's tests on mortar cylinders reported in 1904 to the French Academy of Sciences), on concrete columns reinforced with spiral reinforcement only, and on columns reinforced both longitudinally and transversely (6,7,8,9, and 10). One result of this investigation was the derivation of a simple relationship describing the effect on strength of closely spaced spiral reinforcement. This was

$$f_1 = f''_c + 4.1 f_2 \tag{1.1}$$

where f_1 = unit strength in compression

f''_c = unit strength of "unconfined" concrete

f_2 = average lateral unit stress

For circular reinforcement, the average lateral unit stress was related to the stress in the reinforcement by the following expression

$$f_2 = 2 \frac{A''_s f''_s}{Ds} \tag{1.2}$$

where A''_s = cross-sectional area of transverse reinforcement

f''_s = unit stress in the transverse reinforcement

D = diameter of concrete core enclosed by the transverse reinforcement

s = longitudinal spacing of transverse reinforcement

Equations 1.2 and 1.1 were combined to yield

$$f_1 = f''_c + 8.2 \frac{A''_s f''_s}{Ds} \tag{1.3}$$

or

$$f_1 = f''_c + 2.05 p'' f''_s \tag{1.4}$$

where p" = volume of transverse reinforcement/volume of enclosed concrete.

Although Eq. 1.4 was not offered as a general statement but only as a simplified interpretation of the observed phenomena, it is applicable over a wide range of variables covering almost all possible conditions to be met in practice. Its flexibility and application is hampered by the fact that no intelligible relationship has been offered to determine the stress in the transverse reinforcement, f''_s. This is related to another shortcoming of the interpretations of the test data, the lack of any specific information about the axial deformation of the confined concrete. Although an expression was offered for the strength of the confined concrete, none was given for the deformation at any significant stage of loading.

Considere emphasized the ductility of confined concrete as one of its attractions. He reported a minimum ultimate shortening of 0.02 in axial compression and/or bending and maximum values larger than 0.12 (3). Talbot was also impressed by the toughness of his transversely reinforced columns but, primarily as a result of his test under large repetitive loads where deformations continued to increase at each successive application of load, he thought that the strength of the confined concrete might not be utilized in design as effectively as that of unconfined concrete (11). The test columns of the ACI Investigation also developed strains on the order of 0.02 at maximum load (not at failure). Realization of this guranteed ductility led the ACI Committee 105 to recommending lower factors of safety for spiral columns than for comparable tied columns (5). Unfortunately, this important property of confined concrete was relegated to the background since there was no explicit reference in the form of an algebraic statement to its ductility, as in the case of its strength.

It would appear from the foregoing remarks that two critical decisions were made in the development of the uses of transverse reinforcement: the choice of the circular type as the only type worthy of serious consideration and the choice of strength as the only explicit criterion in design. The first choice restricted indirectly the use of transverse reinforcement to columns. The second choice concentrated the research on strength, with little effort being expended in attempts to define the complete stress-strain curve.

In relation to the present practice in the analysis and design of structures, both of these turns appear to have been taken in the wrong direction. Circular reinforcement was chosen on the implicit assumption that the transverse reinforcement would be used solely to confine the concrete. Emphasis was laid on strength alone, because at the time of the ACI Column Investigation "working stress design" dominated. Today, the rewarding possibility of using limit design for reinforced concrete frames which already have transverse reinforcement as rectangular ties or stirrups is frustrated by the lack of adequate information on the stress-strain characteristics of concrete confined by non-circular transverse reinforcement.

It is possible to salvage from previous investigations some information on load-deformation characteristics of concrete confined by circular transverse reinforcement. However, most of the tests were discontinued after the maximum load was reached. Hence, at best a lower bound to the ductility of confined concrete may be obtained. For the effects of non-circular transverse reinforcement, even this type of information is limited.

Only two experimental investigations aimed directly at obtaining information about the load deformation characteristics of concrete confined by rectilinear reinforcement are known to the writers. One of these was reported by W. W. L. Chan (12) and the results of the other, which was carried out at the Munich Materials Testing Laboratory, have not yet been published.

In the course of an investigation of the rotation capacity of reinforced concrete frame connections, Chan tested eccentrically-loaded prisms and cylinders which were reinforced both longitudinally and transversely. These tests offered a direct comparison between the effects of rectilinear and circular transverse reinforcement. The approximate ranges of variation were 3000 to 5500 psi for the cube strength of the concrete and 1 to 4 percent for the volumetric ratio of the transverse reinforcement. The dimensions of the test specimens were 6 by 6 by 11.5, 6-in. round by 12, and 6 by 3.625 by 52 in.

Chan related the increase in strength of the concrete to the volumetric ratio of the transverse reinforcement using the following expressions.

For circular binding,

$$K_u - K_o = \sqrt{\frac{p_b}{0.0375}} \qquad (1.5)$$

For rectangular binding,

$$K_u - K_o = \sqrt{\frac{p_b}{0.189}} \qquad (1.6)$$

where $K_u - K_o$ = contribution of the transverse reinforcement to the unit strength of the concrete expressed as a ratio of the cube strength.

p_b = volumetric ratio of the transverse reinforcement.

In the interpretation of the test results the effects of the longitudinal reinforcement, the strength and spacing of the transverse reinforcement, and the cross-sectional shape of the prisms were ignored. Equations 1.5 and 1.6 make possible a quantitative comparison between the effects of rectilinear and circular reinforcement. The comparison of the denominators on the right-hand sides of Eqs. 1.5 and 1.6 indicates that, in these tests, the rectangular binding was only about half as effective as the circular binding for the same transverse reinforcement ratio. This quantitative relationship is the same as that observed by Considere.

Chan also derived expressions for the ultimate strain observed in the tests.

For circular binding,

$$\epsilon_u - \epsilon_o = \frac{\sqrt[3]{p_b}}{17} \tag{1.7}$$

For rectangular binding,

$$\epsilon_u - \epsilon_o = \frac{\sqrt[3]{p_b}}{24.5} \tag{1.8}$$

where $\epsilon_u - \epsilon_o$ = increase in ultimate strain made possible by the lateral reinforcement.

Equations 1.7 and 1.8 indicate that the effectiveness of the rectangular binding compared with that of the circular binding was about 70 percent in reference to strain. Chan reported values ranging from 0.015 to 0.025 for circular binding and from 0.012 to 0.018 for rectangular binding.

The tests at Munich were undertaken primarily to investigate the effect of stirrups in ordinary amounts on the resistance of concrete in the compression zone of reinforced concrete beams. The test specimens were eccentrically loaded prisms reinforced transversely (rectilinear stirrups). The series included specimens with and without longitudinal reinforcement. In general, the tests indicated that stirrups caused an increase of concrete strength and ultimate deformation. The addition of longitudinal reinforcement improved the effects of the transverse reinforcement.

On the basis of the available information, it appears that considerable improvements in the ductility of concrete can be effected with the use of rectilinear transverse reinforcement. Although the strength may not be improved as much as in the case of circular transverse reinforcement, the practical reasons for using rectilinear reinforcement may offset this disadvantage.

1.2 Object and Scope

The object of this report is to describe the results of an exploratory series of tests to study the load-deformation characteristics of concrete confined by rectilinear transverse reinforcement.

The test specimens were concrete prisms which measured 25 in. in length and either 5 by 5 or 5 by 10 in. in cross section. All tests were carried out under axial compression. The spacing of the transverse reinforcement was held constant at 2 in. The primary variables were: (1) the amount of transverse reinforcement, (2) the concrete strength, and (3) the shape of the cross section.

1.3 Outline of Tests

A total of 30 specimens were cast and tested in 10 sets of 3. Each set of 3 specimens comprised a prism with no reinforcement, a prism with No. 2 ties spaced 2 in., and a prism with No. 3 ties spaced at 2 in. As indicated in the previous section, all specimens were subjected to axial loading and had the same length, 25 in.

The distribution of the sets according to the two major variables, the cross section and the concrete strength, is summarized in the following table.

Nominal Concrete Strength	Cross Section	
	5 by 5 in.	5 by 10 in.
3000 psi	3	3
5000 psi	2	2

1.4 Designation of the Test Specimens

Each test specimen is designated by four numerals such as 1123. The first numeral indicates the nominal concrete strength, the second numeral the cross-sectional dimensions, and the third numeral the amount of transverse reinforcement. The fourth numeral is used to distinguish the specimens for which the first three variables were the same.

The curve for the first three numerals in the designation is as follows:

The first numeral, 1123

1..... Nominal concrete strength = 3000 psi

2..... Nominal concrete strength = 5000 psi

The second numeral, 1123

1..... 5 by 5-in. cross section

2..... 5 by 10-in. cross section

The third numeral, 1123

0..... No transverse reinforcement

2..... No. 2 ties at 2 in.

3..... No. 3 ties at 2 in.

1.5 Acknowledgments

This work was carried out in the Structural Research Laboratory of the Civil Engineering Department at the University of Illinois while Dr. T. Szulczynski was on leave of absence from the University of Gdansk, Gdansk, Poland, and held a post-doctoral fellowship granted by the University of Illinois.

2. MATERIALS AND FABRICATION

2.1 Concrete

(a) Cement. Marquette brand type III portland cement was used in manufacturing the specimens.

(b) Aggregates. Wabash River sand and gravel were used in all the specimens. Since the 2-in. spacing of the transverse reinforcement left rather small openings on the side of the horizontal form, especially in the specimens reinforced with No. 3 bars, it was necessary to use an aggregate with 3/8-in. maximum size. This aggregate has a smooth rounded surface and results in a ratio of concrete tensile to compressive strength less than that for concretes made with ordinary angular aggregate. The origin of both the sand and the gravel is an outwash of the Wisconsin glaciation. Their absorption was about 1 percent by weight of surface-dry aggregate.

(c) Concrete Mix. Mixes were designed by the trial batch method. The mix proportions for the various sets of specimens are given in Table 1 which also lists the measured slumps, the compressive strength of the concrete indicated by 6 by 12-in. cylinders, and the tensile strength of the concrete measured by splitting tests on 6 by 6-in. cylinders.

The measured initial modulus of deformation of the concrete is compared with concrete strength in Fig. 1. Both quantities are based on tests on 6 by 12-in. cylinders. The plotted data fall considerably below the line representing Jensen's expression (13) for the initial modulus of deformation:

$$E_c = \frac{30 \times 10^6}{5 + \frac{10,000}{f'_c}} \qquad (1.9)$$

where the modulus of deformation, E_c, and the cylinder strength, f'_c, are in psi. The test data are represented better by the broken line in the figure described by a modified form of Equation 1.9, with 7 being used rather than 5 in the denominator. The discrepancy between the plot of Jensen's expression and the data is a result of the properties of the aggregate.

2.2 Reinforcement

Only transverse reinforcement was used in the specimens. The transverse reinforcement consisted of No. 2 and No. 3 ties at a longitudinal center-to-center spacing of 2 in. At each end of the specimen, three ties were placed at a spacing of 1 in. in order to reduce the effect of stress concentrations at the ends.

The No. 2 bars were plain while the No. 3 bars were deformed. The external dimensions of the cold-bent ties were 5 by 5 or 5 by 10 in. The closed tie was formed by lapping a bar at one face for about 2 in. and welding along the length of the lap.

The stress-strain characteristics of the two types of bars used are shown in Fig. 2. The No. 2 bars did not exhibit an ideal "flat-top" region beyond the yield point.

2.3 Fabrication of the Test Specimens

The specimens were cast with their longitudinal axes in a horizontal position. The sides of the forms were made of plastic-treated plywood with a thickness of 1/2-in. and the bottom and sides of the forms were made of 5-in. structural steel channels. The forms were built with extreme care so that the specimens would have the desired dimensions within 1/32 in. The bottoms of the forms were removable; each form could accomodate both the 5 in. sq. and the 5 by 10-in. specimens.

In order to keep the transverse reinforcement in place during the operation of casting, the ties were connected by lengths of soft No. 16 gage wire at each corner. The uniform spacing was maintained by first clamping the stirrups between two pieces of wood and then weaving the soft wire around each stirrup. Although this system provided no lateral bracing, once it was placed in the form it was fairly stiff and it was able to maintain reinforcement in place during vibration.

The ties were arranged so that the laps occurred on two opposite faces only of the specimen and on alternate ties on each face.

The amount of moisture in the sand was determined immediately before casting using the "Speedy" apparatus. Essentially, this apparatus involves the measurement of the increase in pressure in a closed chamber where the wet sand and an amount of sodium carbide are mixed. Previously, the results obtained using this apparatus were checked against oven-dried samples and were found to agree within 1/10 of a percent of moisture content. The gravel was in a bone-dry condition before the mixing operation.

The concrete for all the five sets of specimens having 5-in. sq. cross sections was mixed in a pan type mixer of 2 cu. ft. capacity in batches of 350 lbs. The concrete of two of the sets of specimens having 5 by 10-in. cross sections, sets 12-1 and 12-2, were also mixed in this mixer using two batches of 350 lbs for each set of specimens. The concrete for the rest of the specimens having 5 by 10-in. cross sections were mixed in a nontilting drum mixer of 6-cu. ft capacity.

The concrete was placed in a form with the help of an internal vibrator. The forms were struck 24 hours after casting and the specimens were stored in a moist room (74°F and 100 percent relative humidity) for five days. They were then removed and stored in the laboratory while they were instrumented. The ages of the specimens at the time of test ranged from 8 to 14 days, most of them being tested at either 9 or 10 days as indicated in Table 1.

3. INSTRUMENTATION AND TEST SETUP

3.1 Instrumentation

(a) Measurement of Concrete Deformations. Concrete deformations were measured on the surface of the specimens in both the vertical and horizontal directions with the use of mechanical and electrical strain measuring devices. The basic measuring system consisted of mechanical measurements of the deformation with the use of a 2-in. Whittemore mechanical strain gage.

The locations of the gage plugs are shown in Fig. 3 for the specimens with 5-in. square sections and in Fig. 4 for the specimens with 5 by 10-in. cross sections. In one set of specimens (11-1), the vertical gage lines extended over the middle 18 in. of length of the specimen as indicated in Fig. 3a. In Set 11-2, five gage lines were used in the longitudinal direction as indicated in Fig. 3b. In the other three sets of specimens with 5-in. square cross sections, the basic scheme shown in Fig. 3c was used for the location of the gage line plugs. The arrangement of gage plugs used in the specimens with 5 by 10-in. cross sections are shown in Fig. 4a. In Sets 11-1 and 11-2, a 10-in. Whittemore gage was also used to measure the longitudinal deformation since there was a sufficient number of gage lines.

The gage plugs for the mechanical gages were mounted on the concrete using Eastman 910 cement. The gage plugs measured 3/8 in. in diameter and about 1/4 in. in depth. They were drilled with a No. 48 drill to a depth of 1/16 in. After drilling, the hole was reamed with a 90° punch in order to harden the sides of the hole.

In Specimens 1223, 1233, and 2221, electrical resistance strain gages were used as shown in Fig. 4a. The same arrangement of strain gages was used on the two opposite 10-in. sides of the specimen. The electric strain gages were SR-4 type A3 gages which were 1 in. long with a trim width of 1/8 in. These were mounted using Eastman Kodak 910 cement after the surface of the concrete had been sanded with emery paper and the small holes on the surface of the concrete were filled with an epoxy resin.

(b) Measurements of Deformations in the Transverse Reinforcement. The tie deformations were measured on the outer surface of the ties using a 2-in. Whittemore strain gage. With a No. 48 drill holes to a depth of 1/16 in. were drilled at a spacing of 2 in. on the outer sides of the stirrups, after the specimen had been cast and cured. The location of these holes in the specimens are shown in Figs. 3 and 4. Holes were drilled on two opposite faces of the specimen where there were no lap joints except as indicated in Fig. 4b.

A number of SR-4 type A7 electrical resistance strain gages were used in Specimens 1223, 1233 and 2221 in order to measure deformations on the outer surface of the ties. The location of these gages are shown in Fig. 4. In Specimens 1223 and 1233, electrical strain gages were mounted on both the inside and outside surfaces of the bars. The strain gages inside the bars were mounted prior to casting of the specimens and waterproofed with Epoxoid.

All electric strain gages were mounted using Eastman 910 cement after the surface of the bar had been prepared by sanding with emery paper and cleaning with acetone. In the case of the No. 3 bars, the deformations had to be ground down in order to provide a smooth surface for the application of the strain gage. The gages that were mounted on the outside surface of the bars had no special waterproofing although they were protected by a layer of wax.

(c) Longitudinal Deflection. The longitudinal shortening of the specimens was determined by measuring the distance between the two loading plates at the top and at the bottom. The movement of these two plates with respect to each other was measured at the east and west faces of the specimen using an extensometer equipped with a 0.001-in. dial gage. This measurement was not made for Sets 11-2 and 12-1.

(d) Transverse Deflection of the Ties. The transverse deflection of the middle two or three ties were measured using a C-gage equipped with a 0.001-in. dial. The locations at which the deflections of the ties were measured are indicated in Figs. 3 and 4.

3.2 Test Setup

All specimens were tested in a screw-type testing machine of 300,000-lb capacity. Prior to the series of tests, the stiffness of the machine was measured by extending the legs to a position where they would be in testing the 25-in. long specimens and loading the head of the machine with a hydraulic jack seated on the base. At the expected levels of maximum load, the stiffness of the machine was found to be such that for a movement of 0.01 in. the change in load was about 56,000 lbs. Although the machine was quite stiff, it was not stiff enough not to have the plain concrete specimens fail at or soon after the reaching of the maximum load.

Three different arrangements were used for loading the specimens.

The first two specimens, 1101 and 1121, were supported on 2-in. end plates flush with the base of the testing machine. At the top the load was applied on another 2-in. end plate through a 0.5-in. square bar between the end plate and the head of the machine. This represented a line load applied along one major axis of the cross section. It was used on the assumption that specimens with eccentrically applied loads would be included in the test program. After two tests, it appeared that this type of loading affected the stability of the specimen in advanced stages of straining (about 0.05 strain). Hence, it was abandoned.

Specimen 1131 and Sets 11-2, 12-1, 12-2, 21-1, and 21-2 had the same arrangement at the bottom. At the top, the load was applied on the end plate through a movable head which, ostensibly, did not restrain rotation.

In testing Sets 11-3, 12-3, 22-1, and 22-2 the movable head was fixed after a load of 5000 lb was applied on the specimen.

The different methods of loading resulted in no discernible difference in the measured curves of load versus longitudinal deformation

3.3 Test Procedure

The load on each specimen was increased continually to failure in 1 1/2 to 4 1/2 hours, the duration of test depending on the extent and number of measurements. All measurements were obtained at 10 or 20 kip intervals depending on the capacity of the specimen. Usually it took from 10 to 30 minutes to take one set of readings which were taken in the following order:

1. Applied load.
2. Longitudinal deflection.
3. Electric strain gages (if any).
4. Lateral deflection of the ties.

5. Longitudinal deformation of the ties (transverse to the axis of the specimen).

6. Longitudinal deformations of the concrete.

7. Transverse deformations of the concrete.

8. Repeat measurement of longitudinal deflection.

9. Repeat load measurement.

Since there was serious crushing of the surface concrete at or sometime after the reaching of the maximum load, it was not possible to get reliable measurements from mechanical gage plugs or electric strain gages mounted on the concrete beyond that stage of loading.

4. BEHAVIOR OF THE TEST SPECIMENS

4.1 Discussion of Deformation Measurements

As described in Section 3.1, deformations of the concrete were measured through various means and at various locations. To simplify the discussion of the load-deformation relationships, it is necessary to refer to representative curves and consider them as if they were unique. Therefore, it is desirable first to compare the deformation measurements made at various locations with each other.

The measurement system common to all specimens involved a layout of mechanical strain gage plugs with two vertical and two longitudinal 2-in. gage lengths as shown in Fig. 3c for the 5 by 5-in. specimens and in Fig. 4a for the 5 by 10-in. specimens. The middle line of strain gage plugs was near the midheight of the specimen. The plugs were mounted in this pattern on two opposite sides of the specimen, the longer side being selected in the 5 by 10-in. specimens. In the 5 by 5-in. specimens, the gage plugs were mounted on two opposite surfaces where the ties did not lap.

In Set 11-1, the 2-in. mechanical strain gage layout covered the whole of the two opposite surfaces of the specimens (Fig. 3a). This was done in order to obtain a detailed picture of the strain variation over the surface of the specimen.

Three specimens, 1223, 1233, 2221, were instrumented with SR-4 type A3 electrical resistance strain gages to measure both longitudinal and transverse strains (Fig. 4a).

In all specimens, except those of the first two sets of specimens, the over-all longitudinal deformation of the specimen was measured using deflection dials. Although the readings were not of great importance in the early stages of loading, they were quite useful after spalling of the surface concrete, since the strain gage plugs usually became loose after this stage.

Figure 5 shows the measured deformations of the middle 4 in. of two specimens, one with a 5 by 5 and the other with a 5 by 10-in. cross section. The solid line in the figures show the original position of the gage lines while the broken ones indicate their displaced shapes at two different levels of load. In plotting the deformed shapes, the middle strain gage plug was taken as the datum point and the deformations were plotted to a much larger scale than the base grid.

As would be expected, the deformations indicate a reduction in volume; the longitudinal compressive deformation exceeds the lateral tensile deformation. Furthermore, the relative magnitudes of longitudinal and transverse deformation measured at various locations indicate that these measurements can be averaged in plotting load-deformation curves. Although an eccentricity of load is indicated, it is relatively small. Similar trends indicating eccentricity of load were observed in several other specimens, but in no case was the difference between the two extreme strains greater than 10 percent of the average before the load capacity was reduced considerably.

The variation of axial and transverse strains over the surface of the whole specimen is shown in Fig. 6 for a single load level, 1930 psi. No significant "end effects" were observed over the middle 18 in. of the specimen.

The strain measurements obtained by mechanical and electrical strain gages are compared in Figs. 7a showing the longitudinal strains and Fig. 7b showing the transverse strains. In these figures, strains measured mechanically are plotted against those measured electrically so that a 45° line emanating from the origin would represent complete agreement between the two types of measurements.

The solid curve in Fig. 7a represents the average of the observed relationships between the strains measured electrically and mechanically and the broken lines represent the extreme variations observed. The electrical measurements indicated always smaller strains than the mechanical ones, the absolute difference increasing with increase in load.

Part of the discrepancy between the results of the two types of measurements may be ascribed to time-dependent effects. The electrical gages were read usually about ten minutes before the corresponding mechanical gages. However, at low loads (longitudinal strain less than 0.001), the deflection readings taken immediately after stopping and before resumption of loading indicated average strain increases on the order of the 0.00001. It is extremely unlikely that the corresponding average deformation measured over the 2-in. gages in the middle portion of the specimen would be larger than the over-all average deformation. In any case, even a multiple of the strain 0.00001 does not explain the discrepancy indicated in Fig. 7a.

Since the discrepancy between the readings was noticed while the tests were being carried out, both types of gage systems were checked carefully. The deformations obtained mechanically over 2-in. gage length could be summed to match those measured over longer gage lengths independently. The only confirmation for the performance of the electrical gages was the availability of data from cylinder tests with the same type of concrete on which gages have been mounted with the same procedure. The data from these tests gave excellent correlation between electrical and mechanical measurements. Therefore, it appears extremely unlikely that one set of gages was malfunctioning consistently.

The different gage lengths, 2 in. for the mechanical and 1 in. for the electrical gages, should result in higher strain measurements for the electrical gages but for one detail: the mechanical gages straddled ties while the electrical gages did not. The plotted average deformations in Fig. 6a show that the readings of the 2-in. mechanical gages were fairly uniform throughout the middle 18 inches of the specimen. However, the uniformity of these results does not necessarily imply uniform strains over each gage length. It appears from the comparison of the readings of the electrical gages mounted on the concrete in between the ties and the readings of the mechanical gages which measured the deformation over a 2-in. length including the section with the tie that the longitudinal concrete deformation in the vicinity of the ties was appreciably larger than that existing away from the ties.

Another interesting feature of the curve shown in Fig. 7a is the break occurring at a strain, indicated by the mechanical gages, of about 0.002. In the higher stages of loading corresponding to such strains, the increase in strain which occurred during the recording of readings was on the order of 0.0003. This increase in itself does not explain the break. The most likely cause of this phenomenon is the initiation of very fine cracks in the concrete which resulted in less and less stress being transmitted to the surface concrete in between the ties and more and more of the deformation taking place in the immediate vicinity of the ties as in a chain with weaker links. The appearance of micro-cracks may have interfered with the functioning of the electrical gages. However, it is known from experience with other specimens that the electrical gages working in compression function reasonably well up to strains of 0.004 or even larger.

It was not possible to continue measurements across the 2-in. gage lines after spalling of the surface concrete since the gage plugs were loosened. This took place at a measured longitudinal strain of about 0.0035. Beyond this value, the longitudinal deformations of the concrete had to be based on the measurement of the over-all deflection. The strains based on the over-all deflection were in quite good agreement with the local strain measurements up to spalling of the concrete. Beyond spalling, however, the strain based on the over-all deflection may be less than the maximum local strain since failure was usually localized over a depth of not more than four inches.

As indicated in Fig. 7b, the agreement between mechanical and electrical measurements of transverse strains was very good up to a strain of about 0.0002. Both types of measurements were made at the same level (Fig. 4). After a strain of 0.0002, the readings of the mechanical and the electrical strain gages deviated from each other quite drastically. This was caused by the initiation of hairline cracks in the concrete. Evidently, the appearance of microcracks at or below strains of 0.0002 interfered drastically with the function of the electrical gages while these cracks were sensed only in small uniform increases in deformation by the 2-in. mechanical gages. According to these results, the measurements obtained by the mechanical gages may be interpreted as strains in the concrete up to a strain of about 0.0002. Beyond that strain, they represent average deformations over two inches.

4.2 Relationship Between Load and Longitudinal Strain

Measured load versus longitudinal strain curves are shown in Figs. 8 and 9 for eight of the ten sets of specimens tested. For the first two sets of specimens, 11-1 and 12-1, deflections, and therefore, average strains beyond spalling were not measured. Otherwise, the behavior of these specimens was similar to that of the comparable specimens of Sets 11-2 and 12-2. The load-strain curves and visual observations of the reinforced specimens indicated three general stages of behavior as shown ideally in Fig. 10. The first stage corresponded to the behavior of the plain specimen. Although a nonlinear load-strain relationship was observed within this stage, the total strain increase was small, on the order of 0.0015. The initiation of the second stage was marked by an acceleration in the rate of longitudinal strain. This stage was terminated by severe spalling of the concrete on the surface, a phenomenon which occurred at or immediately after the reaching of the maximum load. Further increase in strain with reduction in over-all resistance of the specimen were the characteristics of the third stage.

The only variable which appeared to affect the first stage was the concrete strength. Increase in concrete strength increased the stress, strain, and stiffness corresponding to the first stage. There was no marked difference between the behavior of plain and reinforced specimens in this stage. Naturally, the plain specimens failed at the end of this range since the testing machine was not stiff enough to prevent the overloading of the specimen once its resistance started decreasing.

In terms of strain, the second stage of loading was initiated at 0.0015 to 0.002 and ended at 0.003 to 0.004, apparently independent of the shape and amount of reinforcement. In some cases, the maximum load was maintained at a strain of 0.005. In general, however, it can be said that the total increase in strain in the second stage was about as much as it was in the first stage. On the other hand, the increase in load was not as much. The increase in load was largest for the 5 by 5-in. specimens with No. 3 bars and smallest for the 5 by 10-in. specimens with No. 2 bars, the relative increase depending on the strength of the concrete.

The parameters to be used in describing the shape of the load-deformation curve in the third stage demand some discussion. In this range, the load resisting capacity of the specimen is reduced. However, this does not mean that the unit stress resisted by intact concrete is decreased. The surface concrete spalls near the end of the second stage. Thus, in computing the average stress in the concrete, the net area (area inside transverse reinforcement) rather than the gross may be used. For the 5-in. square specimens reinforced with No. 3 bars this would result in a 40 percent increase in the nominal stress as compared to the stress based on the gross section. Furthermore, observations of the state of the specimen in this stage of loading indicated the presence of "arches" in the vertical direction spanning between the ties. The height of these arches, indicated by the depth of spalling, appeared to increase as longitudinal deformation was increased, causing a progressive reduction in the cross-sectional area of the intact concrete. In addition, the horizontal deflection of the transverse reinforcement must have created a partial arch in the horizontal plane such that the lateral support system could be visualized as a series of domes spanning between the ties. This effect would tend to reduce the area further. When this is considered in addition to the fact that the strains at the section of failure could have been larger than those indicated by the over-all deflections on which the strains were based in this stage, it appears that the true stress-strain curve might be ascending or at least proceeding at the same level of stress rather than descending as the load-deformation curve does.

In terms of the practical output of a given member, it is the over-all load-deformation curve and not the true stress-strain curve that is significant. In that sense, the plotted curves showing average unit load based on gross section versus average strain do possess significance. However, they are limited in that they refer to the specific conditions of transverse reinforcement size and shape used in the tests.

The direction and extent of the load-deformation curve in the third stage was found to be affected appreciably by the concrete strength, the type of the specimen, and the size of the reinforcement. The downward slope of the curve increased as concrete strength and length of the side of specimen increased and as the size of the reinforcement decreased. For the 5-in. square specimens with No. 3 bars and nominal concrete strength of 3000 psi, the maximum load resisting capacity was maintained at strains on the order of 0.01. On the other hand, for the 5 by 10-in. specimens with No. 2 bars and nominal concrete strength of 5700 psi, the reduction in load beyond the strain of 0.0035 was quite drastic. In fact, it was debatable whether a third stage existed for these specimens.

4.3 Horizontal Strains and Lateral Deflection of the Ties

The measured relationships between the applied load and lateral deformation are summarized in Fig. 11. In this figure the ordinates represent the ratio of the applied load to the maximum load reached and the abscissas the average transverse strain. The data plotted pertain only to the specimens with transverse reinforcement. The solid line represents the average of all the measurements while the two broken lines give the upper and lower bounds to the measurements.

In the early stages of loading, there was little difference between horizontal strains measured at comparable stresses for specimens having the same nominal concrete strength. The measured lateral strain at a stress of about half the compressive strength of the concrete (6 by 12-in. cylinder) was only about 0.0001. The horizontal strains started increasing appreciably at an applied stress equal to about 90 percent of the cylinder strength when the measured horizontal strain was about 0.0002. This load level also marked the initiation of consistently measurable strains in the ties. Under careful examination, longitudinal hairline cracks could be observed in the concrete. As indicated before in the discussion of the relationship between strains measured mechanically and electrically, the average strains shown in Fig. 11 for strains greater than about 0.0002 are not strains but deformations measured over a 2-in. gage length. The abrupt break in the curves in Fig. 11 indicates the initiation of the second stage of loading as defined in the preceding section. The transverse strain readings were not continued beyond the maximum load since spalling of the concrete made the readings of the gages unreliable.

In the initial stages of loading, the ratio of the lateral to the longitudinal strains was 1/8 to 1/6. This ratio started to increase slowly at an applied stress of about half the compressive strength of the concrete. There was a very large increase at about 90 percent of the cylinder strength.

The diagrams shown in Fig. 12 illustrate the lateral deflections undergone by the reinforcement. The data shown in Fig. 12 pertain to the long side of a 5 by 10-in. tie. Deflections were measured at five points as indicated in the figure by the centerlines. The curve for P/A = 3200 psi represents the deflected shape of the stirrup at a load of 94 percent of the maximum reached. At this stage and after maximum load the lateral deflection of the tie was very small. The curve for P/A = 3400 psi represents the deflections of the tie at maximum load. Compared to the condition at 3200 psi, the deflections are more than doubled. Nevertheless, in terms of the absolute length of the tie these deflections are still quite small, in this case on the order of about 15 percent of the diameter and 1 percent of the span of the tie. After the maximum load had been reached, the increase in the lateral deflection of the tie was considerable, in the case shown the deformations reached a maximum of about 0.4 in. However, this was considerably beyond the stage in which the maximum load was reached. The shape of the stirrup after the removal of the load is also shown in the figure. Naturally, after such large deformations the residual deflection was quite large.

In general, the measurements of the deflection of the ties showed that at the time the specimen developed its maximum load, the lateral deflections of the ties were negligible. On the basis of the measured deflection, it was unreasonable to expect much membrane action. Furthermore, the few electrical strain gages that were located on the inside face of the stirrups indicated that at midspan of the tie, the curvature undergone by the tie was not great. On the basis of these measurements, the yield stress existed uniformly throughout the depth of the bar.

5. EFFECT OF TRANSVERSE REINFORCEMENT ON THE STRENGTH OF THE TEST SPECIMENS

The object of this chapter is to present a simple relationship between the observed strength of the test specimen and the amount and configuration of the transverse reinforcement.

The measured ultimate loads for all the test specimens are listed in column 3 of Table 2 in sets of three. The first entry for each set refers to the unreinforced prism. The second and third entries refer to the prisms with No. 2 and No. 3 ties, respectively. The ultimate load is listed also as unit loads based on the gross and net section of the specimen.

To study the influence of transverse reinforcement on a given type of specimen, one could compare the ultimate loads for the three specimens of each set. In general, such a comparison indicates the transverse reinforcement to be rather inefficient. A system of No. 3 ties at 2 in. is very heavy reinforcement, especially for the 5-in. square specimens. Yet, except for Set 11-1, the contribution of No. 3 ties to the strength of the 5-in. square specimens was less than 600 psi in every case. This rating may be satisfactory if the question is limited to finding out what happens to a certain specimen upon the addition of transverse reinforcement. However, it does not reflect the actual effect of the ties on the unit strength of the concrete, and even a crude projection of the information obtained from the tests to other cases requires the use of a reasonable estimate of the true unit strength.

At the time of reaching the maximum load, the surface concrete was spalled. Examination of the specimens after completion of the tests showed that the depth of penetration of the spalling was as much as one inch in some cases. These observations were made after the specimen was strained several times that at maximum load and do not indicate the net section at maximum load.

However, it cannot be denied that the true average stress at maximum load was larger than that based on the gross section. The net section must depend on the longitudinal spacing, the size, and the span of the ties, and the frictional and cohesive properties of the concrete. The concrete must be supported laterally by two perpendicular systems of arches or a system of domes on each side; it spans from tie to tie in the longitudinal direction and most of the reaction must be concentrated near the corners of the tie in the transverse direction. There was definite indication of the arching in the longitudinal direction. The sides of the specimens resembled a washboard after the test. However, there was no direct evidence of arching in the transverse direction.

On the basis of the data available, it would be presumptuous to delineate the shape of the net section with any curve implying precise information. The alternative of using the gross section is equally unattractive since it is definitely wrong. The only practical choice is to define a net section geometrically similar to the gross section. In the quantitative studies made in this chapter, the net section was defined arbitrarily as the area within the tie. It follows from the preceding discussion that the net section should be smaller than this area. However, no plausible defense could be made for a quantitative expression defining the reduction in area intelligibly, on the basis of results from 20 reinforced specimens involving several variables.

The stresses based on this definition of the net section are listed in column 5 of Table 2 for all the test specimens.

In the ACI Column Investigation (6, 7, and 8) it was found possible to express the increase in strength of the concrete resulting from lateral pressure in accordance with the following expression:

$$f_1 = f''_c + k\, f_2 \tag{1.1}$$

where f_1 = unit strength in compression

f''_c = unit strength of the concrete in the specimen without lateral pressure

f_2 = lateral stress

k = a constant derived from test data

In the case of helically reinforced columns, f_2 was expressed as

$$f_2 = \frac{2\, A''_s\, f''_s}{Ds} \tag{1.2}$$

where A''_s = cross-sectional area of the transverse reinforcement

f''_s = stress in the transverse reinforcement

D = diameter of enclosed core

s = longitudinal spacing of the transverse reinforcement

In view of the demonstrated success of Eq. 1.1, it can be assumed that the contribution of the rectilinear transverse reinforcement to the strength of the concrete is independent of the concrete strength and that the factor k is a constant. The definition of f_2 for rectilinear reinforcement is not as simple as in the case of circular reinforcement. The steel stress is not constant along the side of the specimen nor need it be constant over the cross section of the tie. If these variations are ignored for the sake of simplicity, a study of the equilibrium of various sections cut in a lateral slice of a rectilinear core indicates different average stresses acting on different sections. Undoubtedly, shearing stresses exist in the lateral plane over and above those caused by material non-uniformity.

In evaluating the test data, it was assumed that the calculated average stress across a line joining the mid-point of two adjacent sides could be used as a measure of the effect of the transverse reinforcement. In calculating this stress, the yield load of the bar was assumed to be developed. In view of the steel strain measurements at maximum load discussed in Section 4.3, this assumption is not unreasonable. Thus,

$$f_2 = \frac{2\,A''_s\,f''_y}{bs} \cdot \frac{1 + \frac{h}{b}}{1 + (\frac{h}{b})^2} \qquad (5.1a)$$

or

$$f_2 = \frac{p''\,f''_y}{(\frac{b}{h} + \frac{h}{b})} \qquad (5.1b)$$

where A''_s = cross-sectional area of the transverse reinforcement

f''_y = yield stress of the transverse reinforcement

b = width of the enclosed section

h = depth of the enclosed section

s = longitudinal spacing of the transverse reinforcement

p'' = volumetric ratio of the transverse reinforcement

The observed increase in unit strength of the concrete, interpreted as the difference between the measured ultimate load divided by the net area and the unit strength of the plain specimen of the same set, is plotted against the values of f_2 (Eq. 5.1) in Fig. 13. The solid circles refer to medium-strength concrete and the open circles to high-strength concrete.

It is seen that four of the open circles fall considerably above the rest of the data. These open circles represent the data from Sets 22-1 and 22-2. All four specimens considered have 5 by 10-in. cross sections and high-strength concrete.

The unit strength of the concrete from the prisms is compared with that from the cylinders in Fig. 14. There is almost a one-to-one relationship in between the two and all but two points fall on or above the line representing $f_c'' = 0.9\ f_c'$. These two points represent the results from Sets 22-1 and 22-2.

If the strength of the concrete in Sets 22-1 and 22-2 is assumed to be 90 percent of the cylinder strength, the corresponding points in Fig. 13 are modified as shown by the open circles crossed by horizontal lines.

For purposes of comparison, a broken line representing $\Delta f_c = 4.1\ f_2$ is shown in Fig. 13. This line indicates the observed relationship between the lateral stress and the increase of axial stress for circular transverse reinforcement (6, 7, and 8). This comparison is made on the tacit assumption that the lateral stress is defined correctly.

It is seen that the efficiency of rectilinear transverse reinforcement is low compared to circular transverse reinforcement. A reasonable lower bound to the data is represented by

$$\Delta f_c = 1.8\ f_2 \tag{5.2}$$

where Δf_c is the increase in unit strength, while the lower broken line in Fig. 13 described by

$$\Delta f_c = 1.4\ f_2 \tag{5.3}$$

is a lower bound to all the measured values.

The scatter in the data, even without the results of Sets 21-2 and 22-2, is appreciable and raises doubts about the justifiability of selecting the average stress on a line joining the mid-points of two adjacent sides rather than using a direct volumetric ratio, such that

$$f_1 = f''_c + K\,p''f''_y \tag{5.4}$$

where K is a constant to be determined from the data.

This would have been a more reasonable approach if the interpretation of the data had to be limited to a description of the observed phenomena. However, it must be admitted that the shape of the transverse reinforcement affects its efficiency. Hence, although Eq. 1.1 is an over-simplification in itself, the use of Eq. 5.1 which reflects a shape-effect in the expected direction is plausible.

6. SUMMARY

6.1 Object

The object of the exploratory series of tests described in this report was to investigate the effect of rectangular ties on the load-deformation characteristics of concrete. These tests constitute the first stage of an experimental investigation of ties, stirrups, and longitudinal reinforcement on the behavior of concrete in structural members. The ultimate object of the study is to develop intelligible methods for the prediction of the rotation capacity of frame connections in reinforced concrete.

6.2 Scope

A total of 30 horizontally-cast prisms were tested under axial compression. Each prism measured 25 in. in length and had a cross section of either 5 in. square or 5 by 10 in.

The prisms were cast in sets of three, one without reinforcement, one with No. 2, and one with No. 3 ties at 2 in. The ties were welded and placed flush with the form. No longitudinal reinforcement was used. The concrete strength ranged from 2300 to 5700 psi.

The measurements included strains on the surface of the concrete in both principal directions and deformations of the ties.

6.3 Test Results

Curves of load versus longitudinal deformation are shown in Figs. 8 and 9. The behavior of the reinforced specimens was qualitatively similar to that of concrete confined by circular transverse reinforcement and can be idealized into three stages as shown in Fig. 10.

The first stage corresponded to the behavior of the plain specimen. Virtually no deformations were measured in the ties.

The initiation of the second stage was marked by an acceleration in the rate of strain with load. This occurred at a strain of approximately 0.0015 when measurable strains were observed in the ties and the presence of longitudinal micro-cracks in the surface concrete were indicated by the strain gages. This stage was terminated by spalling of the surface concrete, after which the over-all resistance of the specimen started to decrease. At maximum load, the lateral deflection of the ties was small, about 1 percent of the tie span.

Large increases in longitudinal and transverse deformations accompanied by reduction in resistance were the characteristics of the third stage.

Based on a definition of the net section at maximum load as the area enclosed within the tie, the effect of rectilinear transverse reinforcement on the unit strength of concrete in the specimens was expressed as follows:

$$f_1 = f_c'' + 1.8\, f_2 \tag{6.1}$$

and

$$f_2 = \frac{2\, A_s''\, f_y''}{bs} \cdot \frac{1 + \frac{h}{b}}{1 + (\frac{h}{b})^2} = \frac{p''\, f_y''}{(\frac{b}{h} + \frac{h}{b})} \tag{6.2}$$

where f_1 = unit strength in compression

f_c'' = unit strength of prism without reinforcement

A_s'' = cross-sectional area of the transverse reinforcement

f_y'' = yield stress of the transverse reinforcement

b = width of the enclosed section

h = depth of the enclosed section

s = longitudinal spacing of the transverse reinforcement

p'' = volumetric ratio of the transverse reinforcement

The relation of the test data to Eq. 6.1 is shown in Fig. 13.

The ductility of the specimens, especially the rectangular ones with high-strength concrete, was poor. However, the observed behavior of concrete in the compression zone of beams with compression steel indicates that the combined action of longitudinal reinforcement with the ties may improve the ductility considerably.

LIST OF REFERENCES

1. Eddy, H. T. and C. A. P. Turner, "Concrete-Steel Construction; Part I, Buildings," Second Edition, Minneapolis, 1919, Appendix, p. 1.

2. Ransome, E. L. and Alexis Saurbrey, "Reinforced Concrete Buildings," McGraw Hill Book Co., New York, 1912, p. 26.

3. Considere, Armand, "Le Beton Frette et Ses Applications," Publications du Journal Le Genie Civil, Paris, 1907.

4. Considere, Armand, "Experimental Researches on Reinforced Concrete," English Translation by L. F. Moisseiff, McGraw Publishing Co., New York, 1903.

5. "Reinforced Concrete Column Investigation, Tentative Final Report of Committee 105," F. E. Richart, Chairman, ACI Journal, June 1933, Proceedings Volume 29, p. 443.

6. Richart, F. E., A. Brandtzaeg, and R. L. Brown, "A Study of the Failure of Concrete under Combined Compressive Stresses," University of Illinois Engineering Experiment Station Bulletin No. 185, 1928.

7. Richart, F. E., A Brandtzaeg, and R. L. Brown, "The Failure of Plain and Spirally Reinforced Columns in Compression," University of Illinois Engineering Experiment Station Bulletin No. 190, 1929.

8. Richart, F. E., and R. L. Brown, "An Investigation of Reinforced Concrete Columns," University of Illinois Engineering Experiment Station Bulletin No. 267, 1934.

9. Richart, F. E., J. O. Draffin, T. A. Olson, and R. H. Heitman, "The Effect of Eccentric Loading, Protective Shells, Slenderness Ratios, and Other Variables in Reinforced Concrete Columns," University of Illinois Engineering Experiment Station Bulletin No. 368, 1948.

10. Lyse, Inge, "Fifth Report on Column Tests at Lehigh University," ACI Journal, June 1933; ACI Proceedings Volume 29, p. 433.

11. Talbot, A. N., "Tests of Concrete and Reinforced Concrete Columns; Series of 1907," University of Illinois Engineering Experiment Station Bulletin No. 20, 1907.

12. Chan, W. W. L., "The Ultimate Strength and Deformation of Plastic Hinges in Reinforced Concrete Frameworks," Magazine of Concrete Research, Volume 7, No. 21, November 1955 (London).

13. Jensen, V. P., "Ultimate Strength of Reinforced Concrete Beams as Related to the Plasticity Ratio of Concrete," University of Illinois Engineering Experiment Station Bulletin No. 345, 1943.

TABLE 1

PROPERTIES OF THE TEST SPECIMENS AND CONCRETE MIXES

Mark	Width b in.	Depth h in.	Reinf. Bar	Water/Cement Ratio	Mix Proportions C:S:G	Slump in.	Age at Test days	Concrete Strength f'_c psi*	Tensile Strength f_t psi**
1101			-						
1121	5	5	No. 2[a]	0.81	1:3.6:3.9	2.2	10	2550	-
1131			No. 3[b]					(4)	
1102			-						
1122	5	5	No. 2	0.81	1:3.6:3.9	2.8	9	3050	-
1132			No. 3					(5)	
1103			-						
1123	5	5	No. 2	0.81	1:3.6:3.9	6.5	9	3600	350
1133			No. 3					(4)	
1201			-						
1221	5	10	No. 2	0.81	1:3.6:3.9	3.5	8	2300	-
1231			No. 3					(6)	
1202			-						
1222	5	10	No. 2	0.81	1:3.6:3.9	2.0	9	3030	330
1232			No. 3					(6)	
1203			-						
1223	5	10	No. 2	0.81	1:3.6:3.9	1.5	10	3830	300
1233			No. 3					(6)	
2101			-						
2121	5	5	No. 2	0.66	1:2.8:3.0	6.5	9	4610	340
2131			No. 3					(6)	
2102			-						
2122	5	5	No. 2	0.66	1:2.8:3.0	6.5	10	5010	450
2132			No. 3					(6)	
2201			-						
2221	5	10	No. 2	0.66	1:2.8:3.0	3.0	10	4850	330
2231			No. 3					(6)	
2202			-						
2222	5	10	No. 2	0.66	1:2.8:3.0	3.0	14	5690	420
2232			No. 3					(6)	

* Based on 6 by 12-in. cylinders. Numeral in parentheses indicates number of cylinders tested.

** Based on splitting test of 6 by 6-in. cylinder.

a Plain bar, Diameter: 1/4 in., $A_s = 0.05$ sq. in.

b Deformed bar, Nominal Diameter: 3/8 in., Nominal $A_s = 0.11$ sq. in.

TABLE 2

TEST RESULTS

Mark	Cylinder Strength f'_c psi	Meas. Ult. Load P_{ult} kip	$\frac{P_{ult}}{A}$ psi	$\frac{P_{ult}}{A_n *}$ psi
1101		59.5	2380	2380
1121	2550	80	3200	3940
1131		90	3600	4990
1102		80	3200	3200
1122	3050	82	3280	4040
1132		90	3600	4990
1103		85	3400	3400
1123	3600	92	3680	4540
1133		97	3880	5380
1201		123	2460	2460
1221	2300	158	3160	3710
1231		160	3200	4080
1202		160	3200	3200
1222	3030	170	3400	3980
1232		179	3550	4550
1203		170	3400	3400
1223	3830	180	3600	4220
1233		197	3940	5010
2101		110	4400	4400
2121	4610	119	4760	5880
2131		126	5040	7000
2102		115	4600	4600
2122	5010	122.5	4900	6040
2132		128	5120	7110
2201		200	4000	4000
2221	4850	250	5000	5860
2231		255	5100	6500
2202		240	4800	4800
2222	5690	280	5600	6550
2232		290	5800	7400

* A_n = Area inside the transverse ties (Tie diameter taken as 1/4 in. for the No. 2 and 3/8 in. for the No. 3 bars).

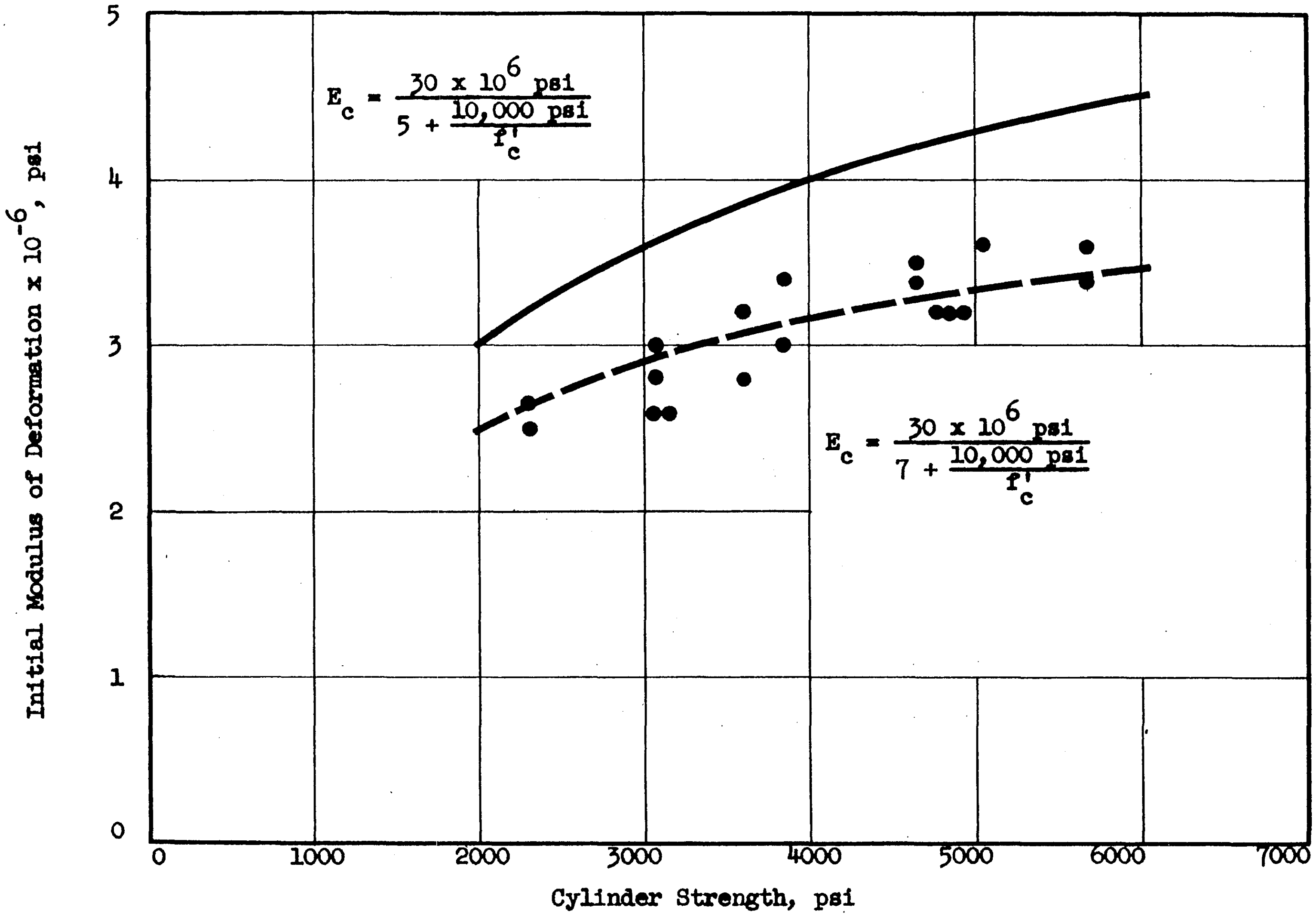


FIG. 1 COMPARISON OF INITIAL MODULUS OF DEFORMATION WITH CYLINDER STRENGTH

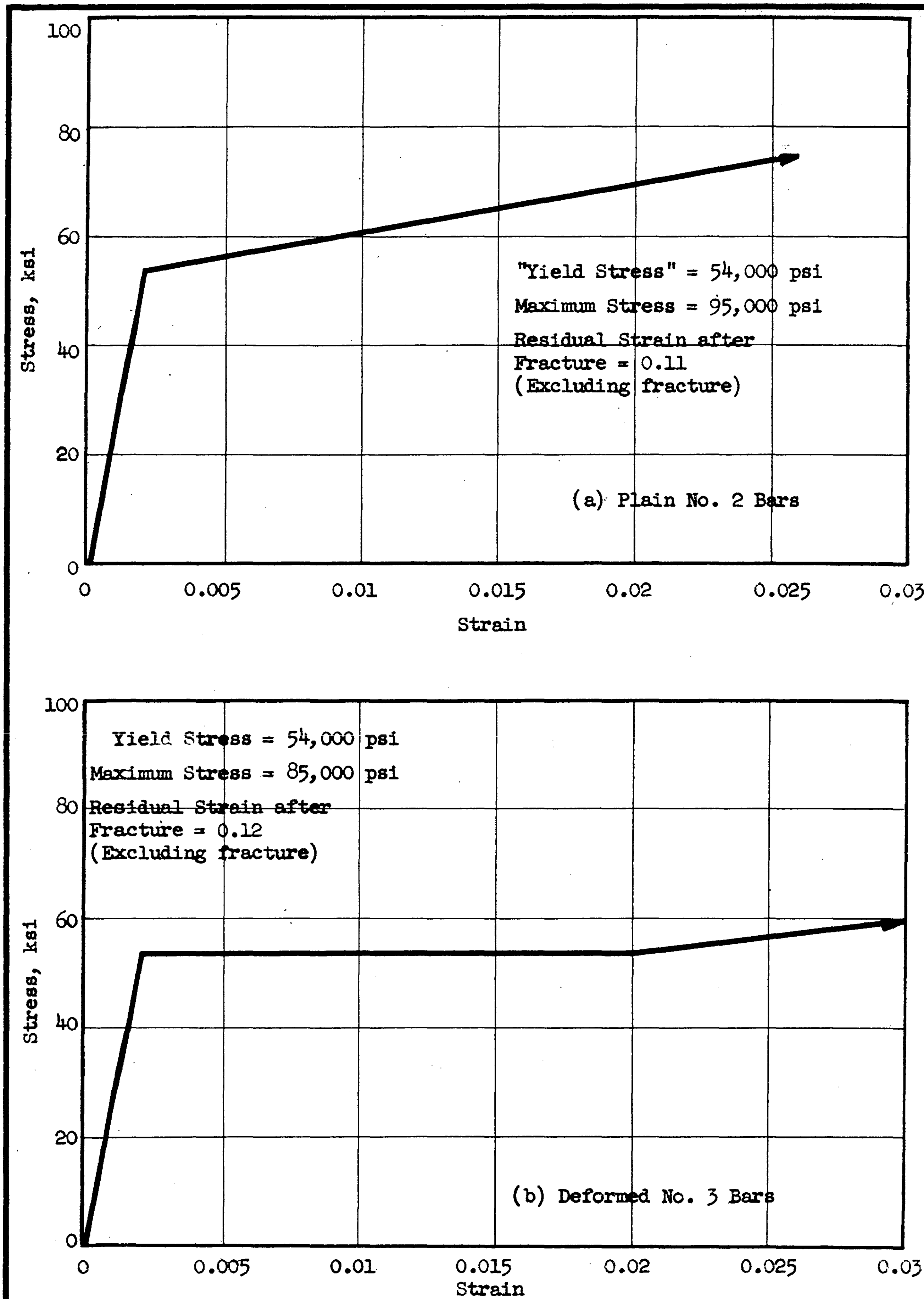


FIG. 2 STRESS-STRAIN CURVES FOR THE REINFORCEMENT

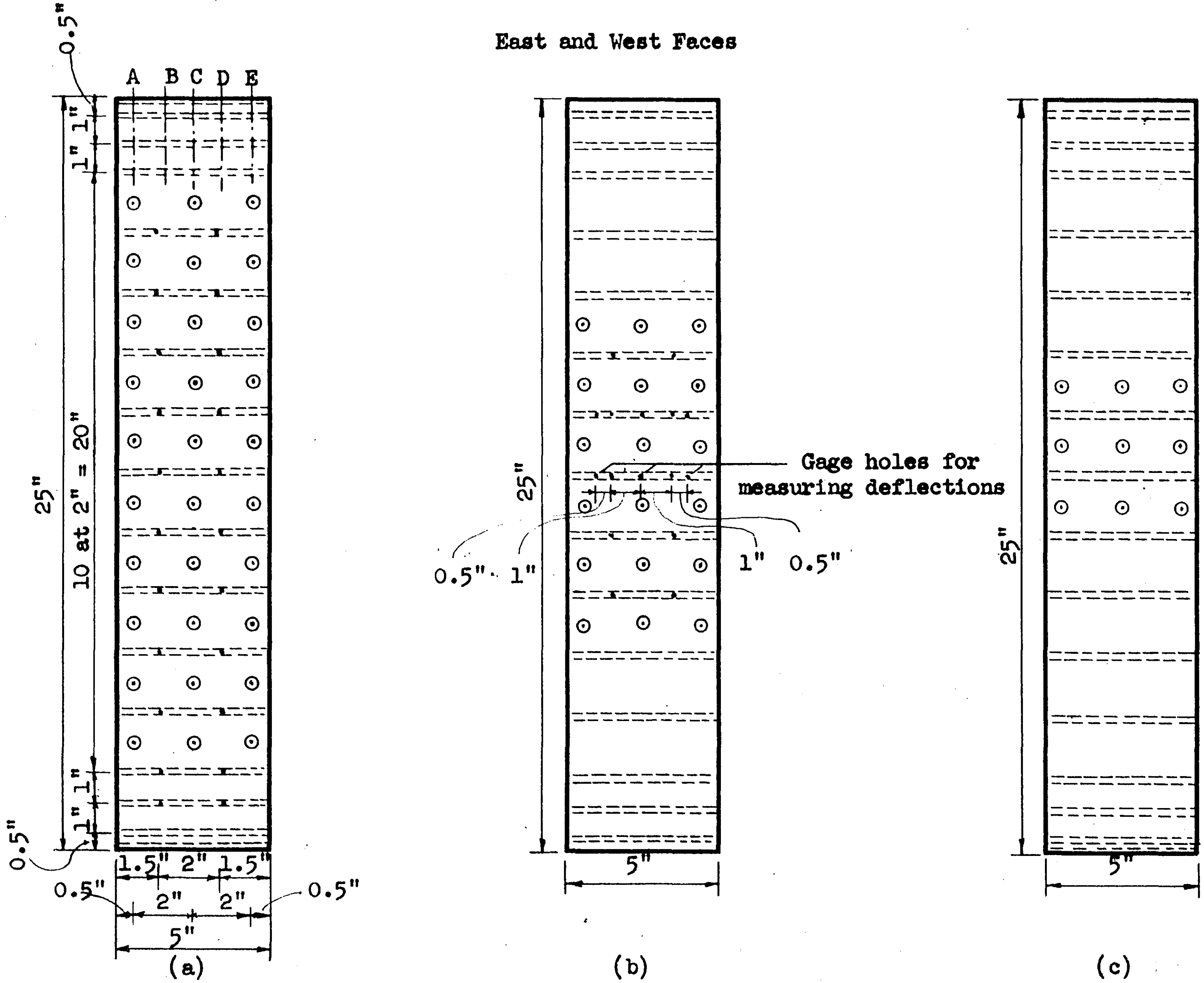


FIG. 3 INSTRUMENTATION FOR THE SPECIMENS WITH 5-INCH SQUARE CROSS SECTIONS

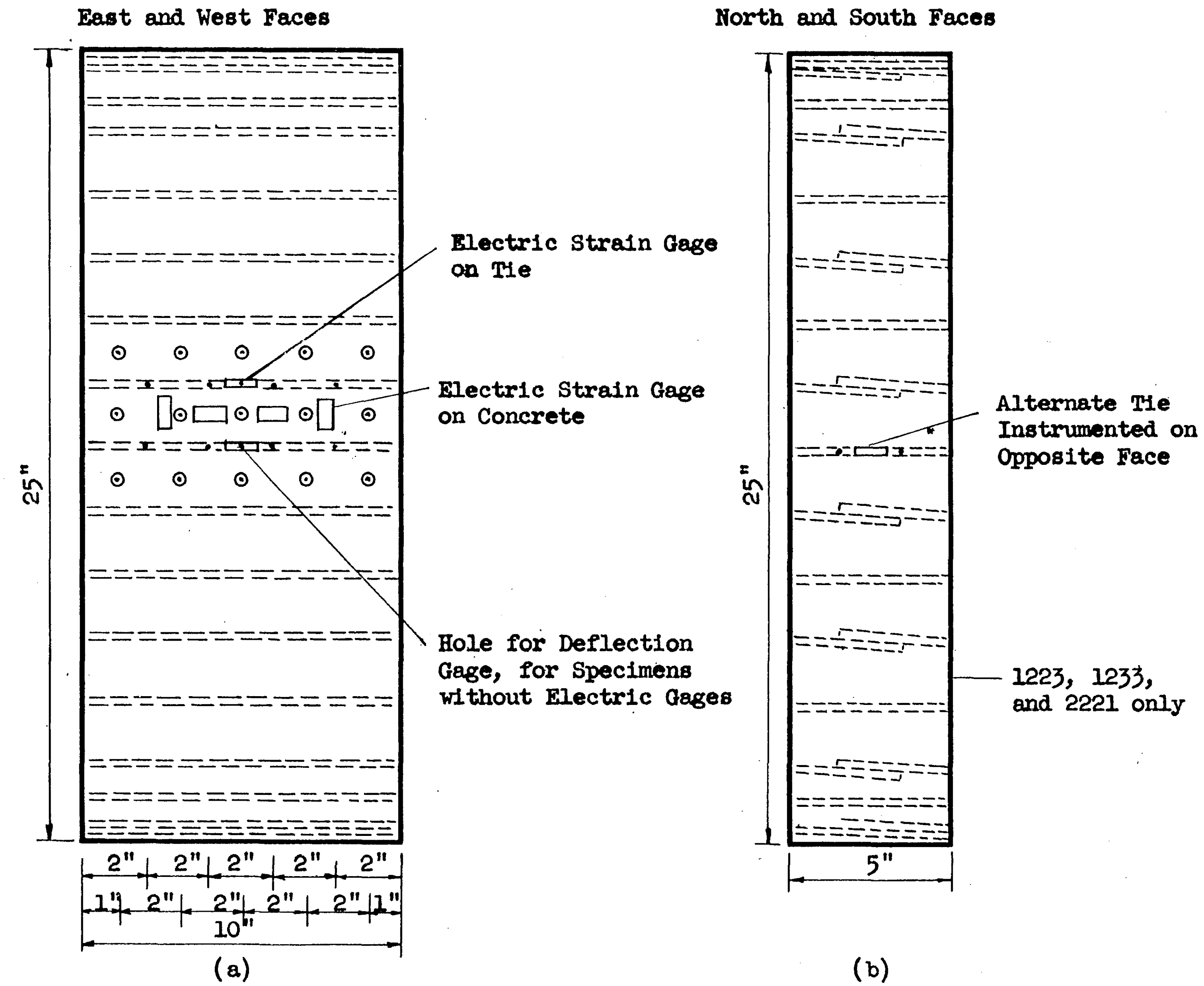
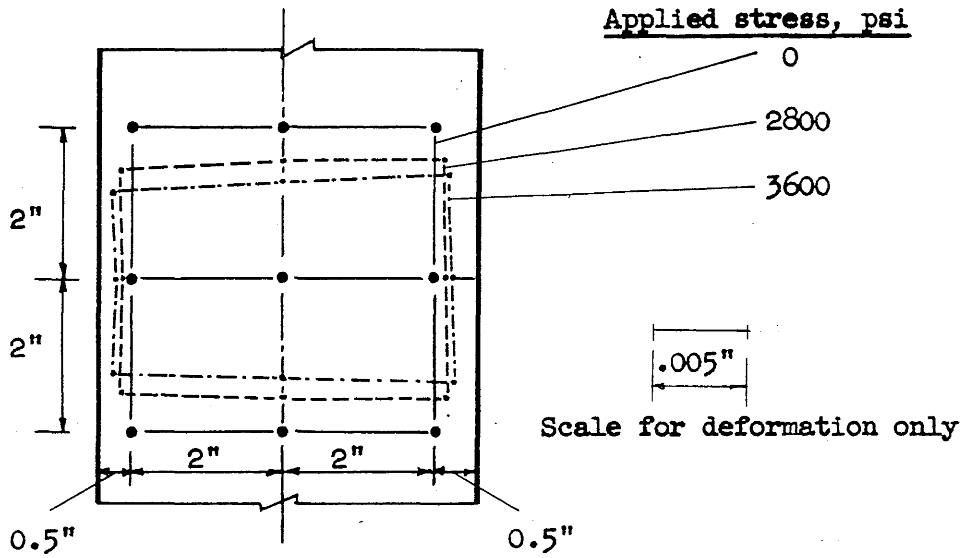
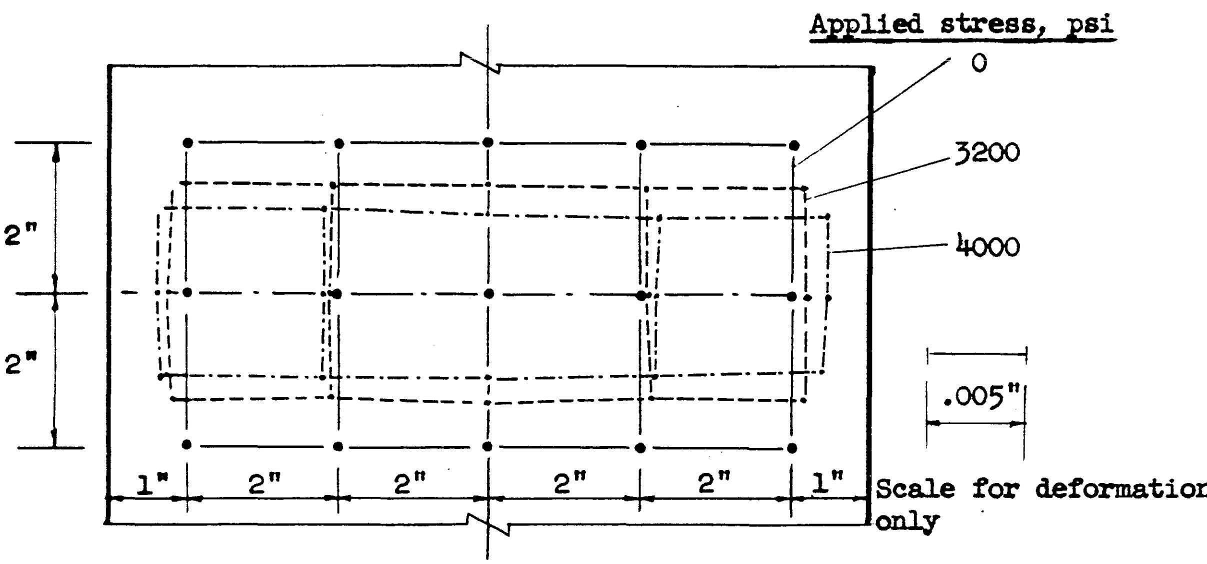


FIG. 4 INSTRUMENTATION FOR THE SPECIMENS WITH 5 BY 10-INCH CROSS SECTIONS

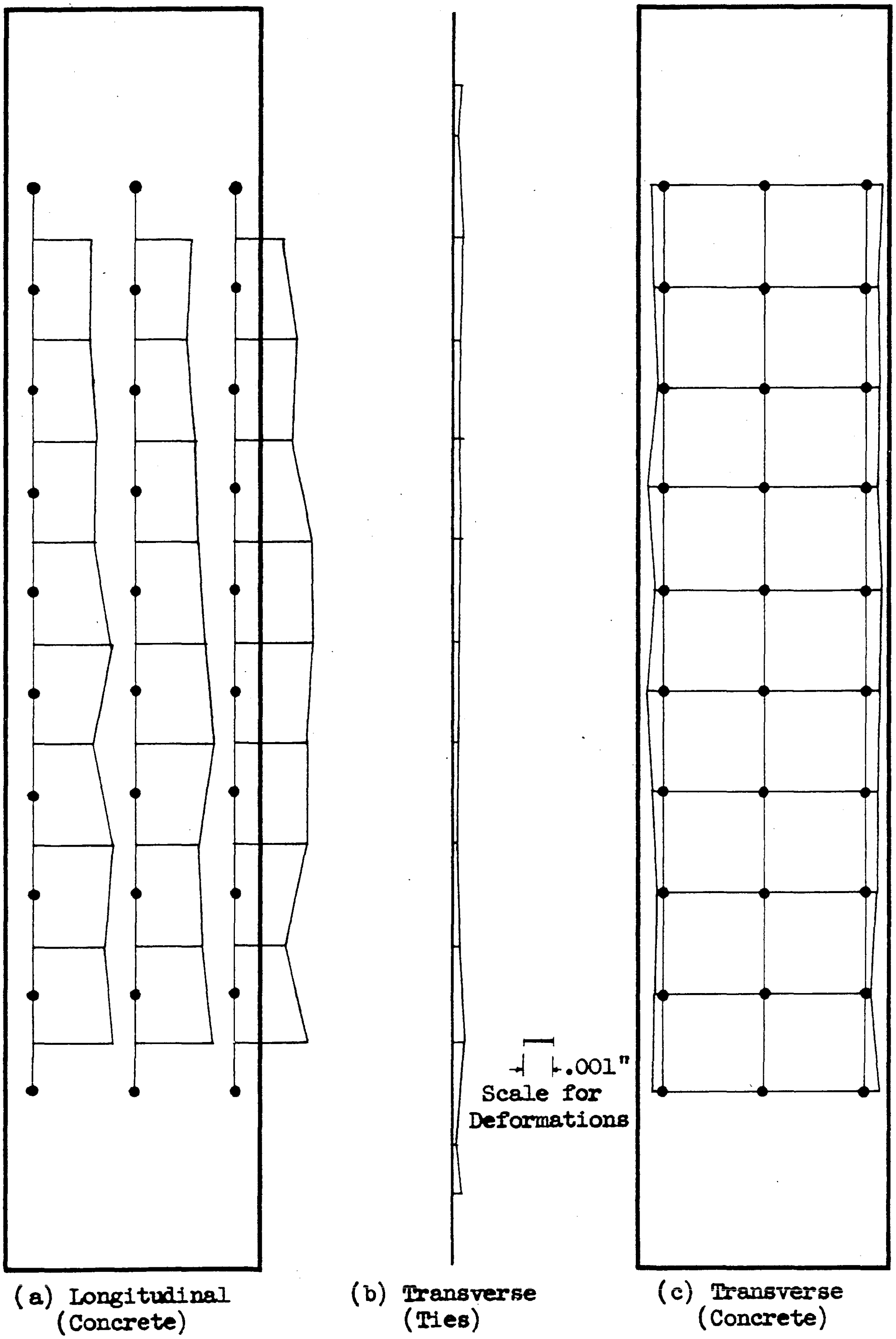


(a) Deformations Measured on East Face of Specimen 2121



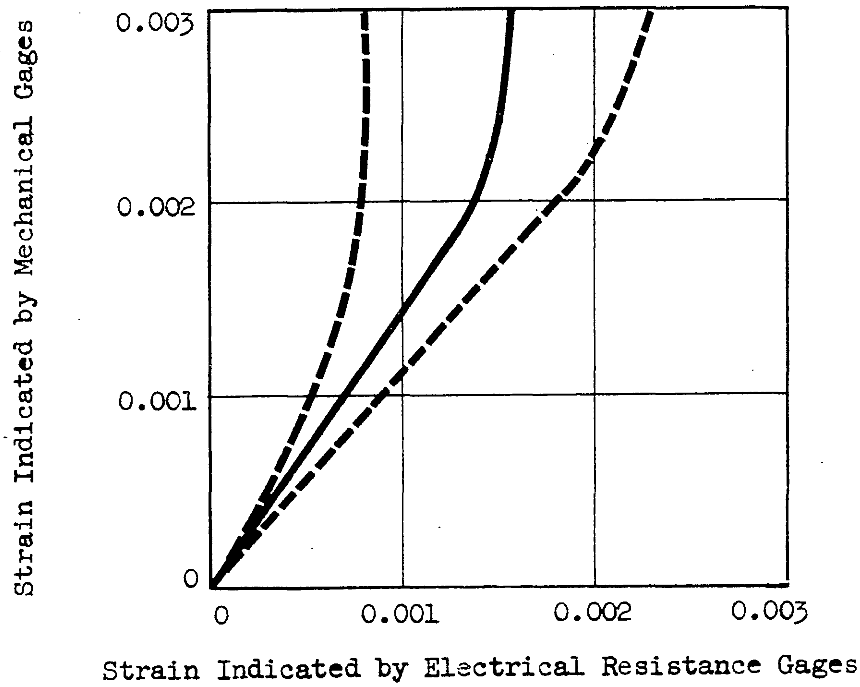
(b) Deformations Measured on East Face of Specimen 2221

FIG. 5 REPRESENTATIVE SURFACE DEFORMATION MEASUREMENTS

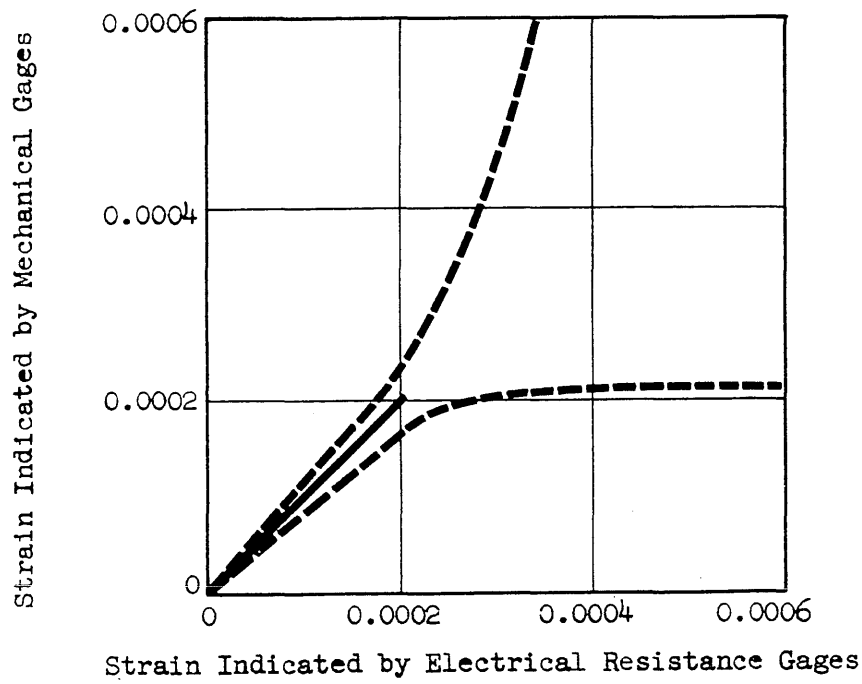


Applied Stress = 1930 psi

FIG. 6 VARIATION OF LONGITUDINAL AND TRANSVERSE DEFORMATIONS IN SPECIMEN 1131



(a)



(b)

FIG. 7 COMPARISON OF STRAINS MEASURED BY ELECTRICAL AND MECHANICAL STRAIN GAGES

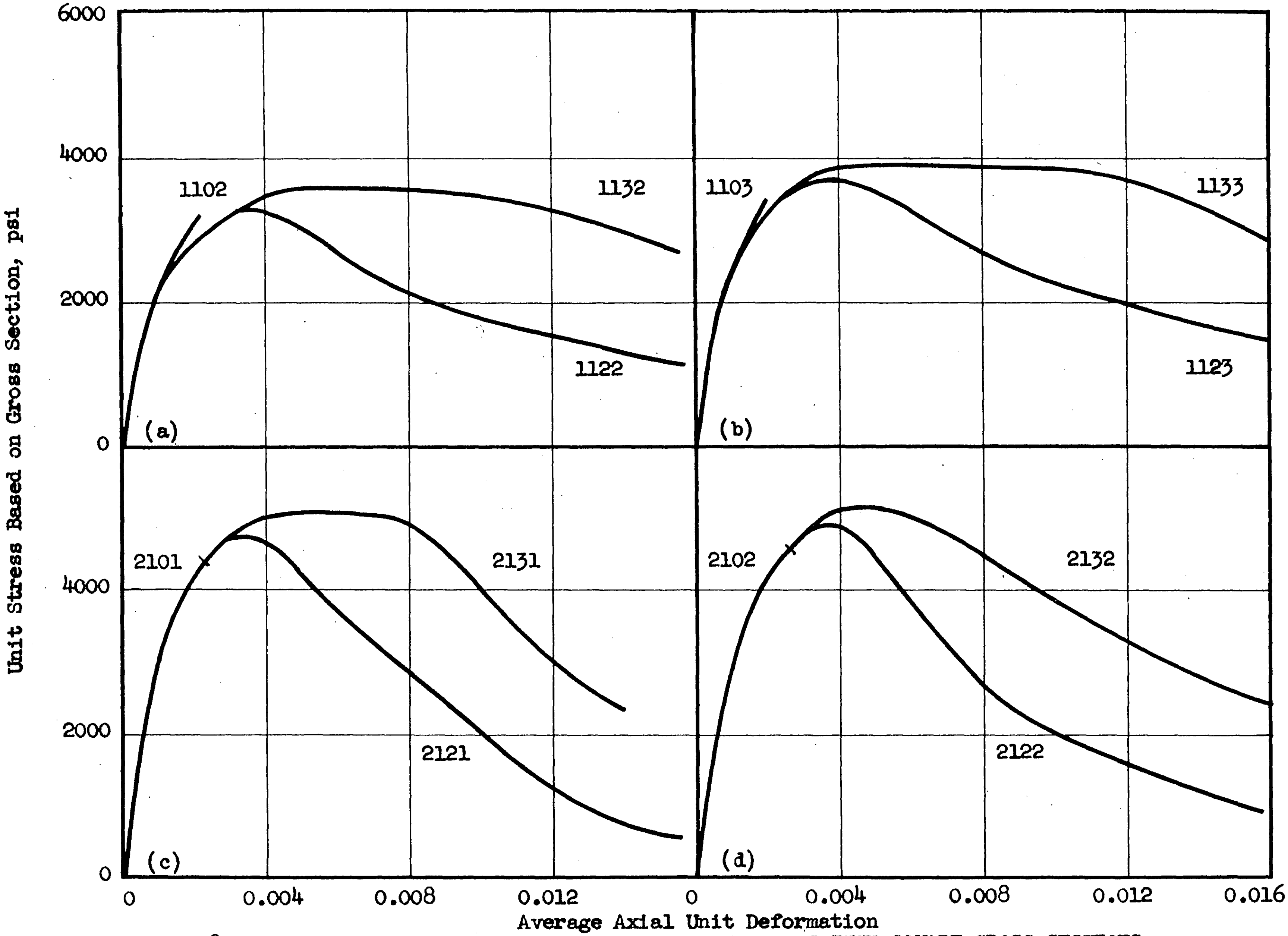


FIG. 8 LOAD-DEFORMATION CURVES FOR THE SPECIMENS WITH 5-INCH SQUARE CROSS SECTIONS

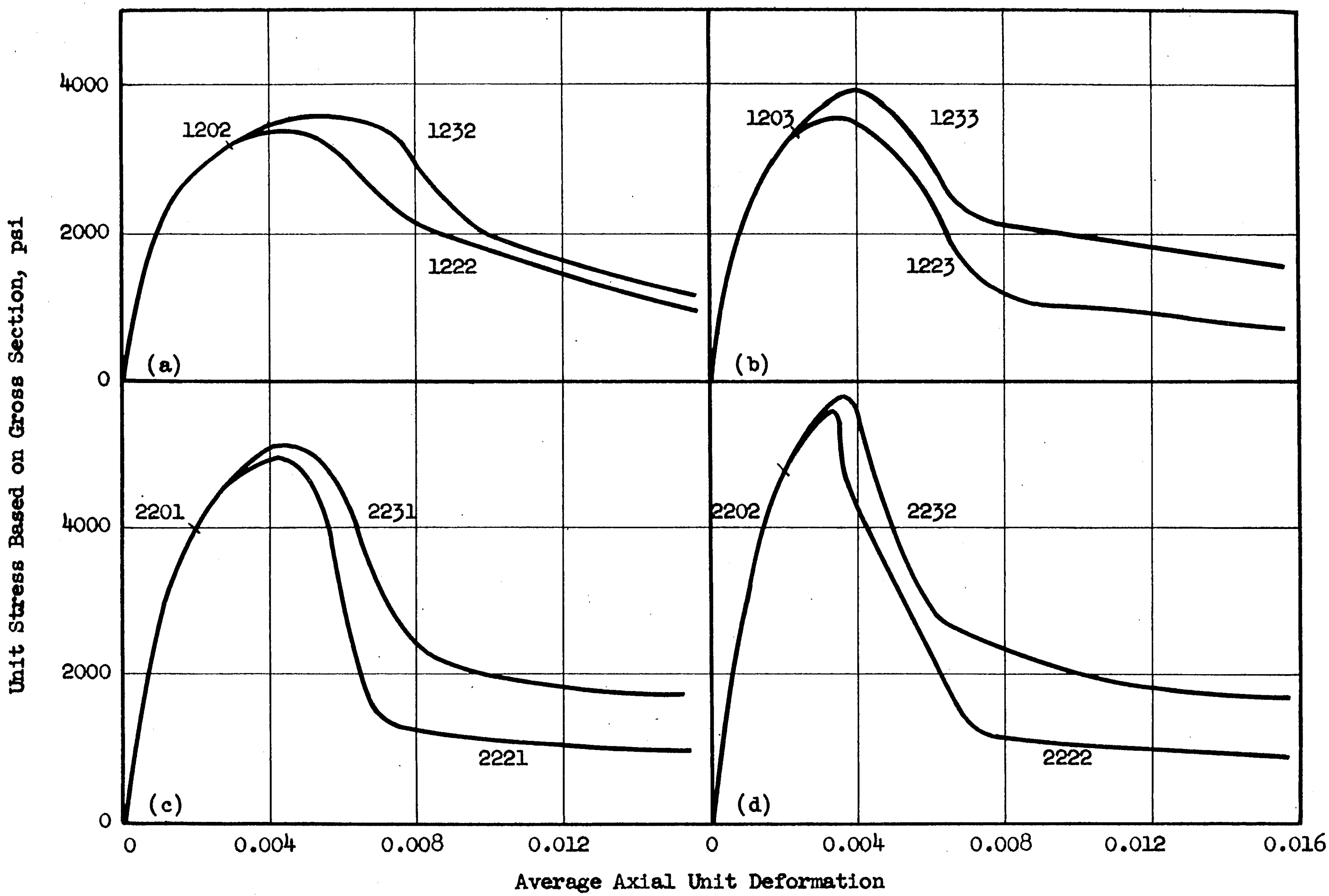


FIG. 9 LOAD-DEFORMATION CURVES FOR THE SPECIMENS WITH 5 BY 10-INCH CROSS SECTIONS

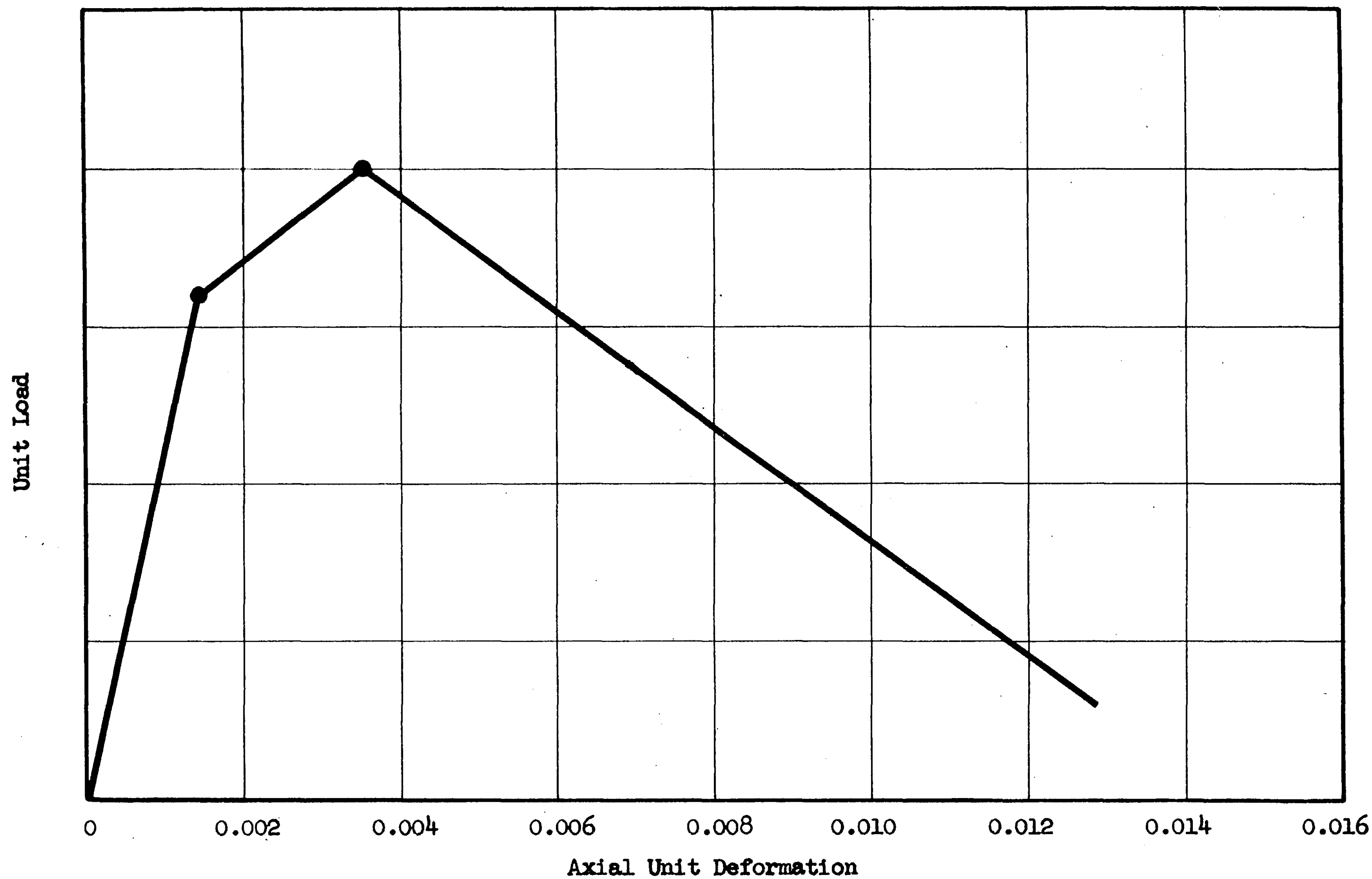


FIG. 10 IDEALIZED LOAD-DEFORMATION CURVE

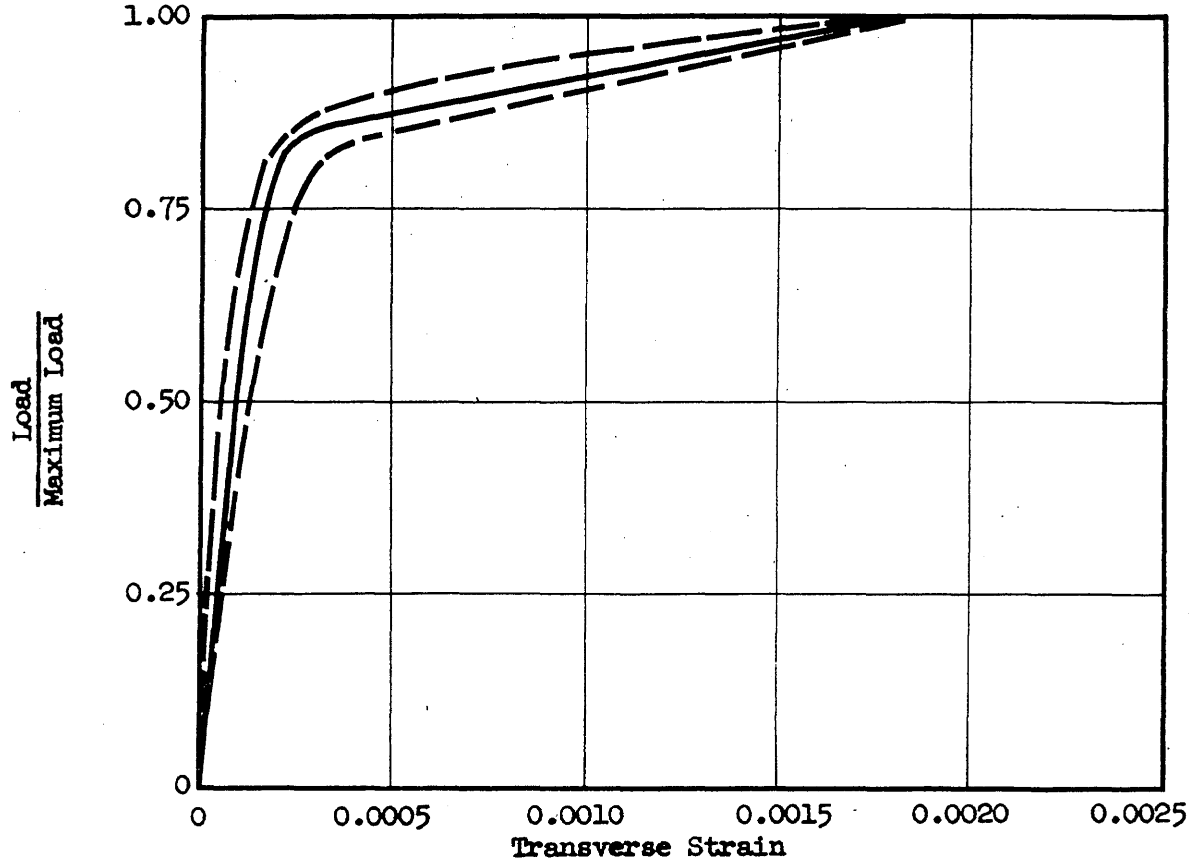


FIG. 11 RELATIONSHIP BETWEEN LOAD AND TRANSVERSE STRAINS

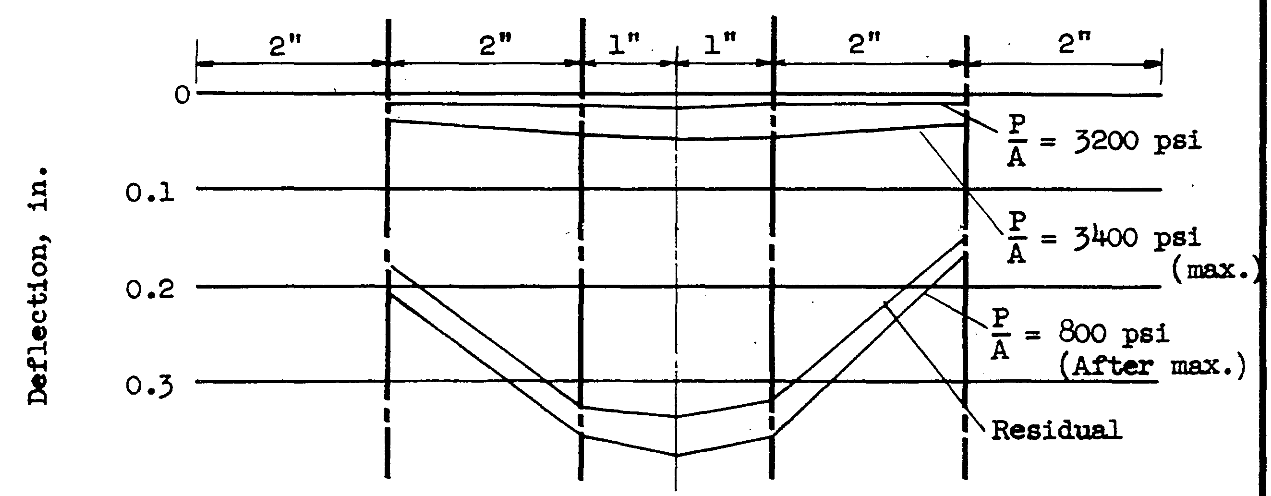


FIG. 12 LATERAL DEFLECTIONS OF A TIE

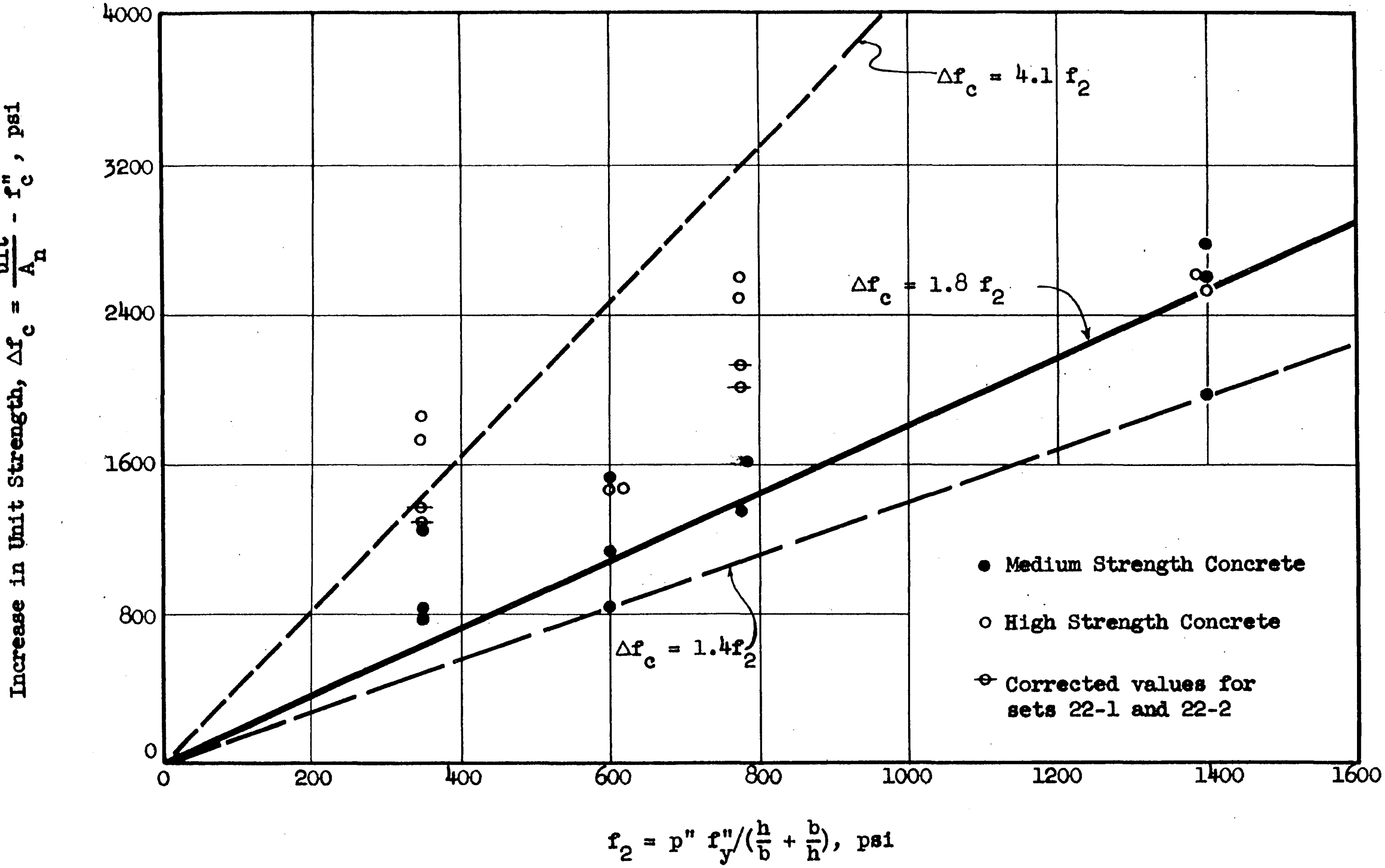


FIG. 13 COMPARISON OF INCREASE IN UNIT STRENGTH WITH AVERAGE TRANSVERSE STRESS

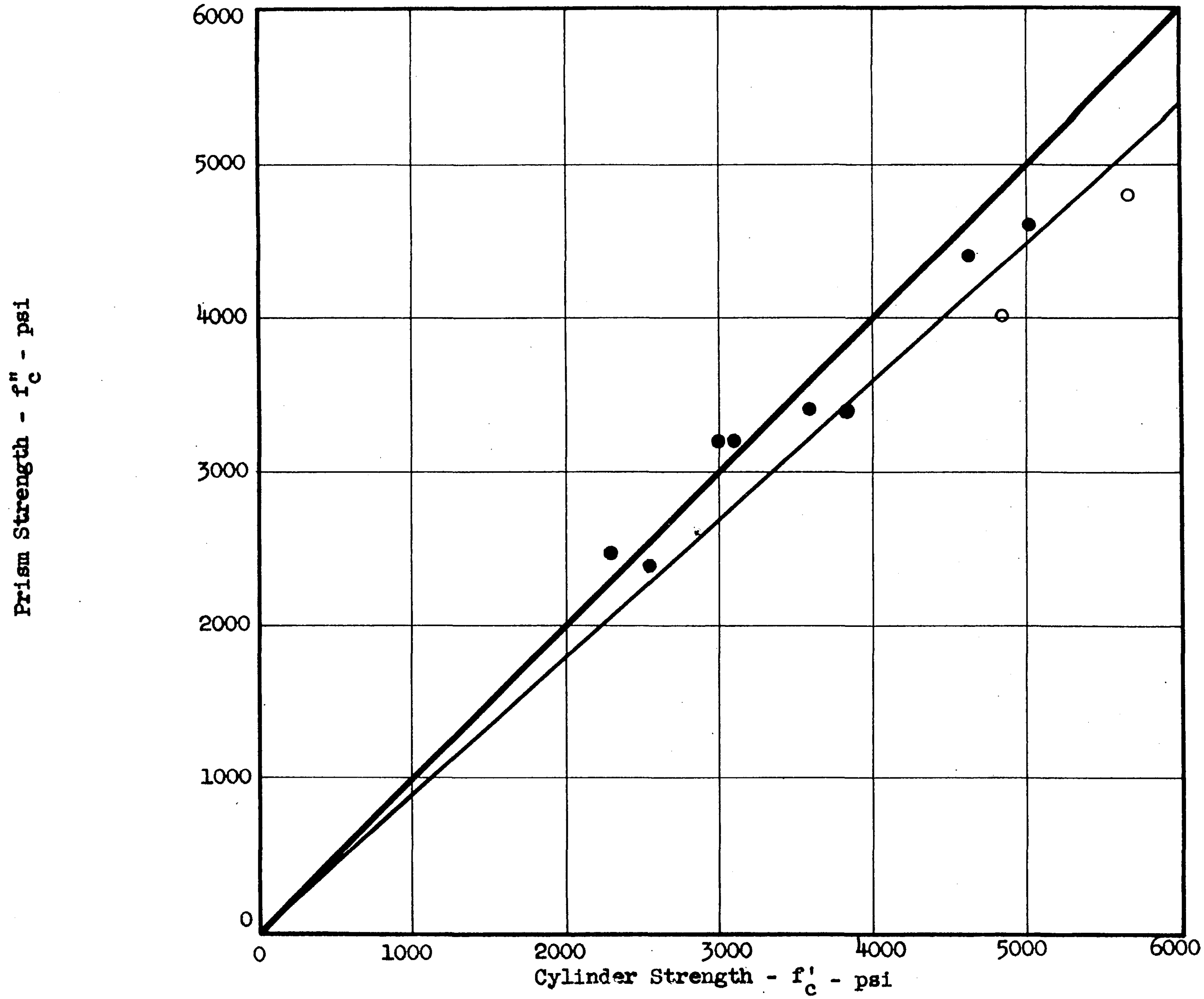


FIG. 14 COMPARISON OF PRISM STRENGTH WITH CYLINDER STRENGTH